Journal of Applied Mechanics and Technical Physics, Vol. 49, No. 1, pp. 84–92, 2008

DETERMINING DYNAMIC CHARACTERISTICS OF MECHANICAL SYSTEMS BY THE METHOD OF CONSTRUCTING ONE-DIMENSIONAL SPECTRAL PORTRAITS OF MATRICES

V. B. Kurzin

UDC 534

A number of important properties of vibrations of linear systems (the quality of stability of the systems, their conditionality with respect to the eigenvalues of the matrices, and the possibility of modeling systems with a large number of degrees of freedom by their subsystems with a smaller number of degrees of freedom), which can be determined by a new mathematical tool called "One-dimensional spectral portraits of matrices" developed under the guidance of S. K. Godunov, are considered. An example is given on constructing one-dimensional spectral portraits for matrices that describe aeroelastic vibrations of hydrodynamic cascades.

Key words: *vibrations, matrix, spectrum, portrait, eigenvalues.*

Introduction. All mechanical systems subjected to vibrational motion possesses certain individual properties responsible for the character of its vibrations under a certain external action. The overall set of system parameters or certain general features associated with these parameters, which describe these properties, characterizes the vibrational motion of the system as part of its dynamic characteristics. In the theory of vibrations, such characteristics are the amplitude-frequency, frequency, and phase characteristics, which can be determined both theoretically and experimentally. Purely theoretical characteristics are discrete spectral portraits of matrices that describe linear vibrations of the systems. The imaginary parts of the eigenvalues of these matrices determine the condition of resonance of forced vibrations of the systems under the action of periodic forces, whereas the real parts determine the damping properties and stability of their vibrational motion.

The characteristics mentioned above, however, fail to describe some important properties of the systems, which determine both stability and resonance conditions. It is known that Lyapunov's theorem determines the character of vibrations of mechanical systems being originally perturbed with respect to the equilibrium position in the asymptotic approximation only and does not evaluate the level of these vibrations in the entire time interval. In addition, the effect of uncertainty in the initial data on the accuracy of determining resonance conditions is not clear. The uncertainty of the initial data may be caused by inaccuracy of theoretical models, technological inaccuracy of manufacturing elements of various structures, and inevitable changes in these structures during their service life. Estimates of the influence of uncertainty of the initial data on the accuracy of determining the dynamic characteristics of mechanical systems is particularly necessary in describing vibrations of non-conservative systems whose matrices are nor normal. There are numerous examples where this influence is fairly significant. In other words, the matrices in the cases mentioned above are ill-posed with respect to their eigenvalues. In particular, Godunov [1] gave an example where the matrix conditionality is practically equal to infinity; for this reason, the accuracy of PC computations is insufficient for eigenvalues to be determined.

The problem of stability and conditionality of arbitrary matrices was comprehensively studied in activities performed in the Siberian Division of the Russian Academy of Sciences and supervised by S. K. Godunov. The

Lavrent'ev Institute of Hydrodynamics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090; kurzin@hydro.nsc.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 49, No. 1, pp. 104–113, January–February, 2008. Original article submitted January 19, 2007; revision submitted March 19, 2007.

 0021-8944/08/4901-0084

results of these activities were summarized in [2, 3]. Based on solving Lyapunov's matrix equation, the maximum estimate of the level of asymptotically stable vibrations in the entire time interval was obtained in [2, 3], an algorithm was developed, and a computational code was written. Concerning the solution of the full problem of eigenvalues, Godunov [2, 3] noted that it is reasonable to study the so-called ε-spectrum, which consists of spots on the complex plane, rather than the precise spectrum. The whole set of these spots forms a two-dimensional spectral portrait of matrices, which provides a clear idea of the accuracy of determining the eigenvalues for a given error in the initial data. It should be noted that the expediency of using the ε-spectrum was simultaneously and independently indicated by Trefethen [4].

For engineering applications, one-dimensional spectral portraits were introduced, an algorithm was developed, and appropriate computer codes were provided. An example of using such portraits for analysis of vibrations of mechanical systems was considered in [5].

The objective of the present work is to describe the properties of system vibrations contained in one-dimensional spectral portraits and not contained in dynamic characteristics known in the theory of vibrations. The possibility of obtaining additional information on the character of vibrations of mechanical systems with the use of one-dimensional spectral portraits of appropriate matrices is demonstrated by an example of the problem of aeroelastic vibrations of hydrodynamic cascades.

1. Spectral Portraits of Matrices. In the theory of vibrations, the spectral portrait of the matrix A that describes the linear vibrations of the system is understood as a set of discrete eigenvalues of this matrix presented in the complex plane. The exact eigenvalues have to satisfy the characteristic equation

$$\det[A - \lambda J] = 0. \tag{1.1}$$

Admitting a certain computational error in determining the eigenvalues and a probable error in the initial data, it seems reasonable to replace Eq. (1.1) by the equality

$$\|(\lambda J - A)^{-1}\| = \infty,$$

where $(\lambda J - A)^{-1}$ is the resolvent of the matrix A.

The eigenvalues of the perturbed matrix $A_1 = A + \Delta$ ($\|\Delta\| \leqslant \varepsilon\|A\|$) belong to the set Λ_ε [$\lambda \in \Lambda_\varepsilon(A)$], which satisfies the condition

$$\|(\lambda J - A)^{-1}\| \leqslant 1/(\varepsilon\|A\|). \tag{1.2}$$

On the complex plane, the set $\Lambda_\varepsilon(A)$ is a multitude of spots of the ε-spectrum, each of them containing one or several eigenvalues of the matrix A_1 satisfying condition (1.2). The boundaries of these spots can be determined as the lines that satisfy the following relation:

$$\|(\lambda J - A)^{-1}\| = 1/(\varepsilon\|A\|).$$

The graphical presentation of spectral spots on the complex plane for different values of ε is called a two-dimensional spectral portrait, which provides complete characterization of localization of matrix eigenvalues for a given matrix perturbation.

Constructing a two-dimensional portrait is a rather complicated computational problem. Therefore, one-dimensional spectral portraits were introduced for engineering applications, which are dependences of norms of the matrices

$$Z_\gamma = \oint_\gamma (\bar{\lambda} J - A^*)^{-1} (\lambda J - A)^{-1}\, d|\lambda| \tag{1.3}$$

on parameters of a certain family of curves that fill the complex plane so that each curve divides the plane and, hence, the spectrum of eigenvalues in terms of a prescribed criterion into two parts (spectrum dichotomy). The norm $\|Z_\gamma\|$ serves as a criterion of the quality of dichotomy of the spectrum of the matrix A by the curve γ.

We assume γ to be a family of straight lines $\lambda = a + it$ and a family of circumferences $\lambda = R\,\mathrm{e}^{i\varphi}$ with varied parameters a and R. Following (1.3), we introduce the matrices

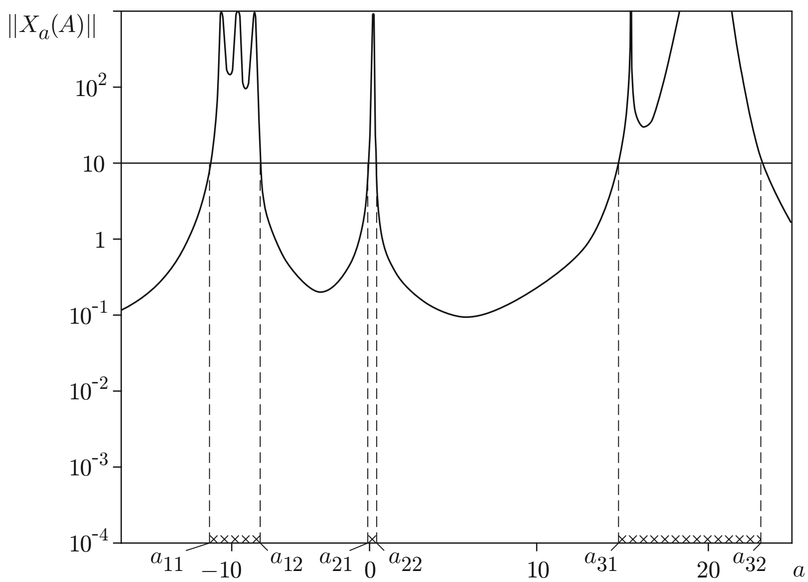

Fig. 1. Spectral portrait of the matrix A [3].

$$X_a(A) = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty}((a-it)J - A^*)^{-1}((a+it)J - A)^{-1}\,dt = \frac{1}{2\pi}\,Z_a(A),$$

$$H_R(A) = \frac{1}{2\pi}\int\limits_{0}^{2\pi}\Bigl(J - \frac{\mathrm{e}^{i\varphi}}{R}\,A^*\Bigr)^{-1}\Bigl(J - \frac{\mathrm{e}^{-i\varphi}}{R}\,A\Bigr)^{-1}\,d\varphi = \frac{R}{2\pi}\,Z_R(A). \tag{1.4}$$

The dependence of the norm $\|X_a(A)\|$ on the parameter a determines the one-dimensional spectral portrait of the real parts of the eigenvalues of the matrix A, and the dependence of the norm $\|H_R(A)\|$ determines the one-dimensional portrait of their absolute values. These matrices satisfy the matrix equations that generalize Lyapunov's equations. In turn, this fact makes it possible to develop algorithms for calculating them with guaranteed accuracy. The resultant matrix equations are solved by the method of orthogonal elimination based on presenting the matrix resolvent in the form of the Lorain series. Prior to this procedure, the matrix is brought to a "cell-diagonal" form. Based on the algorithms developed, a number of codes were designed to compute the dependences of the quality of linear and circumferential dichotomy on various parameters. Figure 1 shows the spectral portrait of the matrix

$$A = \begin{bmatrix} 20 & 14 & & & & & & \\ & 19 & 10 & & & & & \\ & & 20 & 3 & & & & \\ & & & 15 & 1 & & & \\ & & & & 0 & 1 & & \\ & & & & & -7 & 2 & \\ & & & & & & -8 & 4 \\ & & & & & & & -9 \end{bmatrix}$$

borrowed from [3]. As all eigenvalues of this matrix are real, its linear spectral portrait completely characterizes the quality of spectrum dichotomy at given points of the real axis.

2. Some Properties of Vibrations of Mechanical Systems Contained in One-Dimensional Spectral Portraits of Matrices. Let us consider the linear vibrations of the system described by the equation

$$\frac{d\boldsymbol{x}}{dt} = A\boldsymbol{x} + \boldsymbol{f}(\omega), \tag{2.1}$$

where $\boldsymbol{x}$ is the vector whose components are certain generalized coordinates and velocities of their motion, A is the matrix of the coupling coefficients of the components of the vector $\boldsymbol{x}$, and $\boldsymbol{f}$ is the exciting generalized harmonic force with a frequency ω.

In the theory of vibrations, the individual properties of mechanical systems, which determine the level of forced vibrations of the system elements under the action of given forces, are described in terms of the amplitude-frequency characteristics. The conditions of the resonance of forced vibrations and the estimate of stability of free vibrations are contained in the discrete spectral portraits of the matrix A. Let us demonstrate that some important dynamic properties of the system, which are absent in the characteristics indicated above, are contained in one-dimensional spectral portraits. It should be noted that information on the matrix eigenvalues provided by the discrete spectral portrait is also contained in its one-dimensional spectral portraits. Indeed, if the contour of integration γ in Eq. (1.3) passes through the point $\lambda = \lambda_j$ (λ_j are the eigenvalues of the matrix A), then the integrand in Eq. (1.3) has a singularity of the order of the double multiplicity of λ_j. It follows from here that, for $\|X_a\| \to \infty$ and $\|H_R\| \to \infty$, the limiting values of a and R in Eqs. (1.4) determine the real parts and the absolute values of the eigenvalues of the matrix A. The dependence of $\|H_R\|$ on R is qualitatively consistent with the amplitude-frequency characteristic of the system.

2.1. The first property of systems, which is contained in one-dimensional spectral portraits and is not contained in usual dynamic characteristics of systems, is associated with the criterion of stability of their vibrations. Lyapunov's condition, which implies that all eigenvalues of the matrix A have to be located in the left half-plane with respect to the imaginary axis, is the criterion of stability in the asymptotic approximation only, i.e., as $t \to \infty$. Bulgakov [6] obtained an estimate of the form

$$\|\boldsymbol{x}(t)\| \leqslant \|\boldsymbol{x}(0)\| \sqrt{\|X\| \|X^{-1}\|} \exp\{-t(2\|C^{-1}\| \|X\|)\}, \tag{2.2}$$

where the matrix X is the solution of Lyapunov's matrix equation

$$XA + A^* X + C = 0.$$

For a stable matrix A, the absolute value of the function X on its one-dimensional spectral portrait is

$$\|X\| = \|X_a(A)\|, \qquad a = 0.$$

This estimate is important because it contains the property of some systems to excite intense vibrations of individual elements in the course of vibrations treated by Lyapunov's condition as stable at moderate values of the initial data. For instance, systems combining rigid and massive elements with substantially less rigid and massive elements can possess this property. The total mechanical energy of such systems obtained at the initial time may become redistributed during vibrations in the case of certain relations between the elements; as a result, the main part of this energy may be transferred to less rigid and massive elements. Thus, the intensity of vibrations of the latter elements may be significantly enhanced.

2.2. The second property of systems contained in one-dimensional spectral portraits of the matrices A from Eq. (2.1), which describe system vibrations, is the dependence of the frequency and damping characteristics of the system on the magnitude of perturbation of parameters. Determining of this dependence reduces to estimating the conditionality of the corresponding matrices with respect to their eigenvalues.

To estimate the condition number of eigenvalues of the matrix A separated from each other, we consider a perturbed matrix $A_1 = A + \Delta$ ($\|\Delta\| \leqslant \varepsilon \|A\|$). Taking into account Eqs. (1.3) and (1.4), we plot lines parallel to the abscissa axis on one-dimensional spectral portraits of the matrix A, which cross the ordinate axes at

$$\|X_a(A)\| = \frac{1}{\pi\varepsilon\|A\|}, \qquad \|H_R(A)\| = \frac{R}{\pi}\Big[\varepsilon\|A\| + \frac{2}{\pi}\,\varepsilon\|A\|^2\Big]^{-1}. \tag{2.3}$$

The values of a and R at which these lines cross the curves of the dependences of $\|X_a(A)\|$ on a and of $\|H_R(A)\|$ on R are denoted in accordance to the rule

$$a_{11} < a_{12} < a_{21} < a_{22} < \ldots < a_{j1} < a_{j2} < \ldots < a_{n1} < a_{n2}$$

($n \leqslant N$, where N is the order of the matrix).

The segments $[a_{j1}, a_{j2}]$ form one-dimensional spectral spots containing the real parts of the eigenvalues of the matrix A, and the segments $[R_{j1}, R_{j2}]$ form one-dimensional spectral spots containing the absolute values of the eigenvalues. The hatched segments $[a_{j1}, a_{j2}]$ on the abscissa axis in Fig. 1 are spectral spots corresponding to $\varepsilon = 10^{-1}/(\pi\|A\|)$.

Taking into account the additional constructions on one-dimensional spectral portraits considered above, we will prove the validity of the following statement, which characterizes conditionality of the matrix A with respect to its eigenvalues.

Statement 1. *For an arbitrary eigenvalue λ_j of the matrix A, there is an eigenvalue λ_{1k} of the perturbed matrix $A_1 + \Delta$ ($\|\Delta\| \leqslant \varepsilon\|A\|$), the following inequalities being valid for the real parts and absolute values of these eigenvalues:*

$$|\operatorname{Re}(\lambda_j) - \operatorname{Re}(\lambda_{1k})| \leqslant L_{aj}, \qquad ||\lambda_j| - |\lambda_{1k}|| \leqslant L_{Rj} \tag{2.4}$$

($L_{aj} = a_{j2} - a_{j1}$ *and* $L_{Rj} = R_{j2} - R_{j1}$).

To prove this statement, we use the inequalities that follow from Theorem 4.2 in [3]

$$\max_t \|[(a+it)J - A]^{-1}\| \leqslant \pi\|X_a(A)\|,$$

$$\max_\varphi \|(R\,\mathrm{e}^{i\varphi}\,J - A)^{-1}\| \leqslant \frac{\pi}{2R}\sqrt{\|H_R(A)\|^2 + \frac{4}{\pi^2}\,\|H_R(A)\|}, \tag{2.5}$$

and also the inequality

$$\|(A - \lambda J)^{-1}\| \geqslant 1/(\varepsilon\|A\|), \tag{2.6}$$

which has to be satisfied by the eigenvalue λ of the matrix A_1 in accordance with the definition of two-dimensional spectral spots. With allowance for Eq. (2.3) and under the condition

$$a \notin L_{aj}, \qquad R \notin L_{Rj} \tag{2.7}$$

the following inequalities hold:

$$\|X_a(A)\| < \frac{1}{\pi\varepsilon\|A\|}, \qquad \|H_R(A)\| < \frac{R}{\pi}\Big[\varepsilon\|A\| + \frac{2}{\pi}\,(\varepsilon\|A\|)^2\Big]^{-1}.$$

It follows from here that inequalities (2.5) and (2.6) are incompatible under condition (2.7), i.e., they are satisfied only for $a \in L_{aj}$ and $R \in L_{Rj}$. The statement is proved.

To describe the property of the matrices characterizing their conditionality with respect to the eigenvalues, we present the quantities L_{aj} and L_{Rj} in inequalities (2.4) at fixed values of $\varepsilon = \varepsilon_0$ in the form

$$L_{aj} = k_{aj}\varepsilon_0, \qquad L_{Rj} = k_{Rj}\varepsilon_0. \tag{2.8}$$

Note, in the general case, the coefficients k_{aj} and k_{Rj} depend on ε. For eigenvalues, each being located in the corresponding spot of the ε_0-spectrum, the following presentation is valid for $\varepsilon < \varepsilon_0$:

$$k_{aj} = c_{0j} + c_{1j}\varepsilon + c_{2j}\varepsilon^2 + \ldots, \qquad k_{Rj} = d_{0j} + d_{1j}\varepsilon + d_{2j}\varepsilon^2 + \ldots. \tag{2.9}$$

According to the method of perturbations, the coefficients k_{aj} and k_{Rj} in Eqs. (2.8) with allowance for Eq. (2.9) for disparate eigenvalues yield estimates for the corresponding components of eigenvalues with accuracy to the first-order parameter ε_0 in the entire range of the parameter $\varepsilon < \varepsilon_0 \ll 1$. In the terminology of the theory of perturbations, these coefficients are called the condition numbers of the matrix A.

Nevertheless, if the spot of the ε-spectrum contains several eigenvalues (such a spot is usually called a cluster), presentation (2.9) for the corresponding coefficients of expressions (2.8) is incorrect, because the spot may become split as the parameter ε decreases with respect to ε_0 (see Fig. 1). In the general case, the coefficients k_{aj} and k_{Rj} in Eqs. (2.8) characterize the conditionality of the matrix A with respect to the corresponding components of eigenvalues only for fixed values of the parameter $\varepsilon = \varepsilon_0$.

A certain feature in spectral portraits, which is important for practical applications, should be noted: conditionality of close eigenvalues contained in one cluster is usually substantially worse than the conditionality of eigenvalues with no other eigenvalues in the corresponding cluster. It is next to impossible to distinguish eigenvalues contained in one cluster. In this case, the vibrations in the neighborhood of the resonance have an unpredictable set of the corresponding modes.

2.3. The third property of the system, which allows identification of the radial spectral portrait of the matrix A, is the dependence of the basic modes of system vibrations on various generalized coordinates. If some basic modes of system vibrations depend only weakly on some generalized coordinates, the latter may be skipped. In this case, the study of vibrations of the initial system in terms of the corresponding modes can be approximately reduced to studying vibrations of a system with a smaller number of degrees of freedom. In other words, it becomes possible to model a mechanical system described by a high-order matrix as systems with lower-order matrices. For this property to be manifested, the matrix A is brought to the “cell-diagonal” form prior to the development of algorithms for constructing one-dimensional spectral portraits. According to this form of matrix presentation on the spectral portrait, spectrum decomposition into clusters occurs. Each cluster corresponds to a cell-submatrix, which may be used to model the spectral portrait of the full matrix at the corresponding section of the cluster. The accuracy of modeling these submatrices is determined by the depth of the valley between the clusters in the dependences of $\|H_R(A)\|$ on R.

3. Application of One-Dimensional Spectral Portraits for Determining the Characteristics of Aeroelastic Vibrations of Hydrodynamic Cascades. Let us consider bending and torsion vibrations of blade cascades in a gas flow. For a two-dimensional model of a geometrically homogeneous cascade, small bending and torsion vibrations of the blades are described by the system of differential equations

$$\begin{gathered} m\ddot{h}_n + S_\alpha \ddot{\alpha}_n + K_h h = qbL_n + cbF_n, \\ S_\alpha \ddot{h}_n + J_\alpha \ddot{\alpha}_n + K_\alpha \alpha_n = qb^2 M_n \qquad (n = 1, 2, \ldots, N), \end{gathered} \tag{3.1}$$

where $h_n(t)$ is the translational displacement of the characteristic cross section of the nth blade owing to its bending, $\alpha_n(t)$ is the angle of turning of the cross section of the nth blade with respect to a certain fixed point, m and J_α are the mass per meter and the moment of inertia of the cross section with respect to the elastic axis, K_h and K_α are the coefficients of bending and torsion rigidity, respectively, S_α is the static moment of the blade cross section with respect to the elastic axis, N is the number of blades in the cascade, q is the dynamic pressure of the incoming flow, b is the chord of the blade cross section, and c is the factor of rigidity. The right sides of the equations of this system determine the unsteady aerodynamic forces and moments acting on the blades and the forces of elastic bonding between the blades.

According to the theory of cascades in an unsteady flow [7], the dimensionless quantities L_n and M_n determining the aerodynamic forces and moments acting on the nth blade can be presented as

$$L_n = \sum_{r=1}^{N} \Big(l'_{r-n,h} \frac{h_r}{b} + l''_{r-n,h} \frac{\dot{h}_r}{\omega b} + l'_{r-n,\alpha} \alpha_r + l''_{r-n,\alpha} \frac{\dot{\alpha}_r}{\omega} \Big),$$

$$M_n = \sum_{r=1}^{N} \Big(m'_{r-1,h} \frac{h_r}{b} + m''_{r-n,h} \frac{\dot{h}_r}{\omega b} + m'_{r-n,\alpha} \alpha_r + m''_{r-n,\alpha} \frac{\dot{\alpha}_r}{\omega} \Big).$$

Here ω is the eigenfrequency of cascade vibrations in an empty space; the coefficients at h_r, $\dot{h}_r$, α_r, and $\dot{\alpha}_r$ responsible for aerodynamic interaction of the blades are given in [8].

The forces F_n of elastic bonding of the blades in the model considered are determined by the formula

$$F_n = (h_{n-1} + h_{n+1} - 2h_n)/b.$$

We introduce the dimensionless variables

$$\gamma = \frac{S_\alpha}{mb}, \quad \eta = \frac{S_\alpha}{J_\alpha}, \quad \nu = \frac{\omega_\alpha^2}{\omega_h^2}, \quad \varepsilon_0 = \frac{q}{m\omega^2}, \quad \sigma = \frac{c}{m\omega^2}, \quad z_n = \frac{h_n}{b} \tag{3.2}$$

($\omega_h^2 = K_h/m$ and $\omega_\alpha^2 = K_\alpha/J_\alpha$ are the squared partial eigenfrequencies of bending and torsion vibrations of the blades in an empty space, respectively).

With allowance for Eqs. (3.2), system (3.1) is transformed to

$$\begin{gathered} \ddot{z}_n + \gamma \ddot{\alpha}_n + \omega_h^2 z_n = \omega^2 (\varepsilon_0 L_n + \sigma F_n), \\ \ddot{\alpha}_n + \eta \ddot{z}_n + \omega_\alpha^2 \alpha_n = \omega^2 \varepsilon_0 M_n \qquad (n = 1, 2, \ldots, N). \end{gathered} \tag{3.3}$$

If we write system (3.3) in the form of $2N$ ordinary differential Hamilton equations of the first order, we can present the system in the form (2.1).

To calculate one-dimensional spectral portraits of the matrix A, we used the following quantities affecting the aerodynamic influence coefficients as the initial parameters: parameters γ, η, ν, σ, and ε_0, which determine the rigidity and mass characteristics of the blades and aerodynamic and elastic coupling of their vibrations; geometric parameters of the cascade (solidity τ, stagger angle β, and bending of the mid-line of the blade profile normalized to the chord f); and also the Strouchal number $k = \omega b/V$.

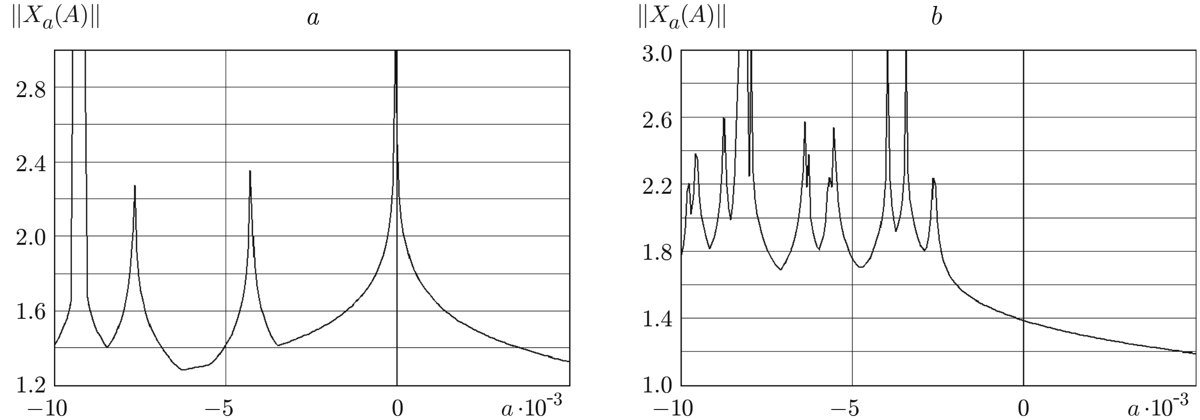

Fig. 2. Dependence of $\|X_a(A)\|$ on a for $\nu = 2$, $\sigma = 0$, and $\gamma = \eta = -0.003$ (a) and -0.3 (b).

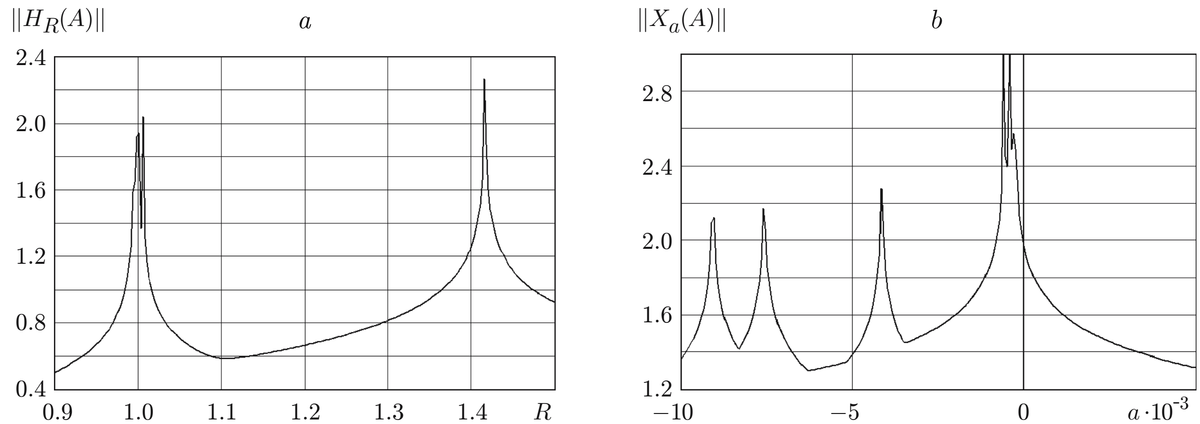

Fig. 3. Dependences of $\|H_R(A)\|$ on R (a) and of $\|X_a(A)\|$ on a (b) for $\gamma = \eta = -0.03$, $\nu = 1$, and $\sigma = 0$.

Figures 2–5 shows the one-dimensional spectral portraits of the matrices that describe vibrations of a cascade of thin blades in an ideal incompressible fluid flow with $N = 10$, $\tau = 1.5$, $\beta = 30°$, $f = 0.025$, $k = 0.5$, $\varepsilon_0 = 0.01$, and varied values of the parameters γ, η, ν, and σ.

Figures 2a and 2b show the linear spectral portraits of the matrices for $\nu = 2$, $\sigma = 0$, and $\gamma = \eta = -0.003$ and -0.300, respectively. By comparing the values of $\|X_a(A)\|$ for $a = 0$ in Figs. 2a and 2b, we see that the quality of stability of aeroelastic vibrations of the blades depends substantially on the positions of their elastic axes with respect to the center of gravity, as is predicted by Eq. (2.2). This conclusion agrees with the known law for flutter characteristics of the blades.

Figures 3 and 4 show the one-dimensional spectral portraits of the matrices for $\gamma = \eta = -0.03$, $\nu = 1$, and $\sigma = 0$ (variant 1) and for $\gamma = \eta = 0.03$, $\nu = 1$, and $\sigma = 0.02$ (variant 2), respectively. A comparison of the radial portraits of the matrices shows that the conditionality of these matrices with respect to their eigenvalues is better in the first variant (Fig. 3a) than in the second variant (Fig. 4a), as is predicted by Eq. (2.4). Considering the matrix of aerodynamic influence coefficients as the perturbed component of the full matrix A, we can conclude that the perturbations of eigenvalues (including their real parts) are greater in the second variant than in the first one. By comparing Figs. 3b and 4b, we see that the second variant corresponds to a smaller reserve of stability of the system.

The property of matrices, which allows the system with a greater number of degrees of freedom to be modeled by its subsystems with a smaller number of degrees of freedom, is described by the one-dimensional spectral portraits shown in Fig. 5. Figures 5a and 5b show the one-dimensional spectral portraits of the matrix corresponding to the

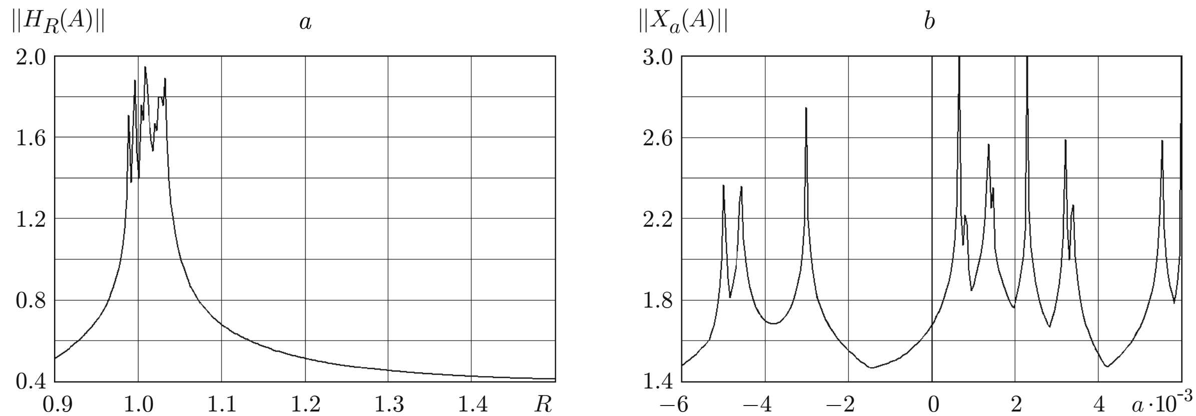

Fig. 4. Dependences of $\|H_R(A)\|$ on R (a) and of $\|X_a(A)\|$ on a (b) for $\gamma = \eta = 0.03$, $\nu = 1$, and $\sigma = 0.02$

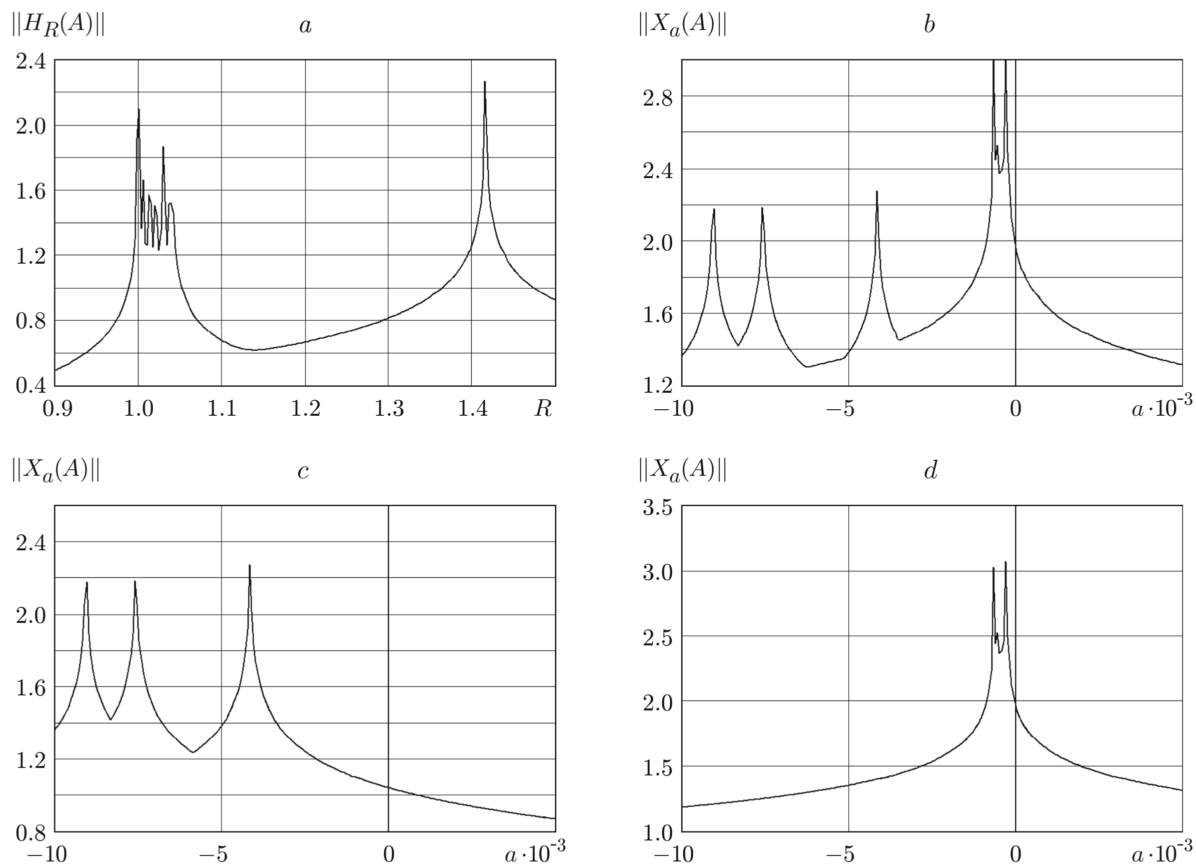

Fig. 5. Dependences of $\|H_R(A)\|$ on R (a) and of $\|X_a(A)\|$ on a (b–d) for the full matrix (a and b) for $\gamma = \eta = -0.03$, $\nu = 2$, and $\sigma = 0.02$ and for its submatrices (c and d) corresponding to clusters of the spectral portrait shown in Fig. 5a.

system that has the parameters $\gamma = \eta = -0.03$, $\nu = 2$, $\sigma = 0.02$, and dimension $N_1 = 20$. The radial spectral portrait of this system (Fig. 5a) reveals two clusters with a rather deep valley between them. According to the third property (see Sec. 2.3), the initial system can be modeled by two subsystems with the dimensions $N_1 = 10$. Figures 5c and 5d show the linear spectral portraits of the corresponding submatrices, which are in good agreement with the portrait in Fig. 5b.

Conclusions. It is demonstrated in the present work that the new mathematical tool "One-dimensional spectral portraits of matrices" developed under supervision of S. K. Godunov can be used to calculate the dynamic characteristics of mechanical systems, which cannot be determined by other methods at the moment.

The author is grateful to S. K. Godunov for his attention to the present work and useful comments and to A. A. Saitgalin for his assistance in computations.

This work was performed within the Integration Project Nos. 5 and 2.12 of the Siberian Division of the Russian Academy of Sciences.

REFERENCES

1. S. K. Godunov, *Ordinary Differential Equations with Constant Coefficients* [in Russian], Vol. 1, Izd. Novosib. Univ., Novosibirsk (1994).
2. S. K. Godunov, *Advanced Aspects of Linear Algebra* [in Russian], Nauchnaya Kniga, Novosibirsk (1997).
3. S. K. Godunov, *Lectures on Advanced Aspects of Linear Algebra* [in Russian], Nauchnaya Kniga, Novosibirsk (2002).
4. L. N. Trefethen, "Pseudospectra of matrices," in: D. F. Griffiths and G. A. Watson (eds.) *Numerical Analysis*, Longman (1991).
5. S. K. Godunov, V. B. Kurzin, V. G. Bunkov, and M. Sadkane, "Application of a new mathematical tool "One-dimensional spectral portraits of matrices" to the problem of aeroelastic vibrations of turbine-blade cascades," in: *Turbomachines: Aeroelasticity, Aeroacoustics, and Unsteady Aerodynamics*, Torus Press Ltd., Moscow (2006), pp. 9–23.
6. A. Ya. Bulgakov, "Effectively calculated parameter of the quality of stability of systems of linear differential equations with constant coefficients," *Sib. Mat. Zh.*, **21**, No. 3, 339–347 (1980).
7. D. N. Gorelov, V. B. Kurzin, and V. E. Saren, *Aerodynamics of Cascades in an Unsteady Flow* [in Russian], Nauka, Novosibirsk (1971).
8. D. N. Gorelov, V. B. Kurzin, and V. E. Saren, *Atlas of Unsteady Aerodynamic Characteristics of Cascades* [in Russian], Nauka, Novosibirsk (1974).